Combustion Instability in Solid Propellant Rockets

The following 6 papers dealing with Linear Acoustic Instability were presented at the AIAA Solid Propellant Rocket Conference in Palo Alto, California, January 29–31, 1964 Additional papers on Nonlinear Acoustic Instability and on Nonacoustic Instability will be published in the July issue

Linear Acoustic Gains and Losses in Solid Propellant Rocket Motors

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The general significance of acoustic gains and losses to the problem of rocket motor stability is discussed briefly Recently, considerable progress has been made toward the understanding of some of the important loss mechanisms These are noted and attention is called to their significance With respect to acoustic gains, the burning surface of the propellant is recognized to be a principal source of acoustic amplification, and a considerable amount of experimental and theoretical attention has been devoted to the determination of the propellant response function It is recognized that, in view of the highly complicated nature of the phenomena, one should hardly expect quantitative agreement between theory and experiment Nevertheless, one should be entitled to expect agreement of a qualitative natue whenever the suppositions of the theory are reasonably well met by the conditions of experiment A comparison of the kinds of results obtained suggests that such a qualitative similarity does not always exist, particularly for low burning rates As a result, it seems likely that some mechanism, which can be important under these conditions, has been omitted from consideration in the existent theoretical model Theoretical calculations, as yet unpublished, indicate that under some conditons radiation may exert an important influence on the acoustic response of the burning surface, especially at low burning rates (Radiative effects have been omitted in previous theoretical treatments) The general nature of the new calculations and some of the numerical results are discussed

Nomenclature

c = $[\gamma \bar{P}/\bar{\rho}_b]^{1/2}$
C = heat capacity of solid propellant
C_p = constant pressure heat capacity of propellant gas
f = frequency, sec^{-1}
j = conditioning temperature sensitivity index for steady-state burning $[= (T/\bar{m})(\partial \bar{m}/\partial T_c)_{\bar{P}}]$
k = radiation absorption coefficient
m = mass flow rate per unit area
n = pressure sensitivity index for steady state burning $[= (\bar{P}/\bar{m})(\partial \bar{m}/\partial \bar{P})_T]$
P = pressure
q_0 = radiative heat flux incident on solid
r = burning rate
T = temperature
v = velocity
γ = specific heat ratio
δ_0 = perturbation parameter $(\sigma_* T_b^4/mC_pT_0)$
ϵ = dimensionless incremental pressure amplitude
λ = wavelength of isentropic sound field
λ = heat conductivity of solid
μ = dimensionless incremental mass flow rate
ρ = density (ρ_s = density of solid)
σ_* = Stefan-Boltzmann constant $[1\ 37(10)^{-12}$ cal/cm^2 sec-K$^4]$
τ = characteristic time
Ω = dimensionless angular frequency $(\omega\bar{\rho}/\bar{k}\bar{m})$
ω = angular frequency
$(\bar{\ })$ = mean value
$(\tilde{\ })$ = Fourier component, i e , F = real $(\tilde{F}e^{i\omega t})$
$(\ )_b$ = mean conditions in burnt gas far from surface
$(\ )$ = mean conditions in the solid far from surface
$(\ )_0$ = solid gas interface

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I Introduction

THE purpose of the present paper is twofold First, it is to provide a context to show how the several papers that follow fit into the general problem of linear acoustic instability of solid propellant rocket motors Second, it is to present some of the recent Applied Physics Laboratory work that deals with the subject of acoustic gains and losses in such motors

In order to be able to design a solid fuel rocket motor successfully, one needs to know that the propellant will burn stably in a steady and predictable fashion Unfortunately, the fact that a propellant may burn stably in one particular motor is no assurance that it will do so in another, for stable burning is not, in general, determined exclusively by either the propellant or the motor configuration

The primary concern of linear theory is whether or not the ever present arbitrarily small pressure fluctuations in a rocket motor will grow in amplitude It is widely recognized that questions of the growth or decay of pressure disturbances fall naturally within the domain of acoustics, so that the stability question is most appropriately posed in acoustic terminology One visualizes the rocket motor as an acoustic cavity, having a variety of resonant frequencies, at which it will tend to be most easily excited Whether or not small disturbances will succeed in exciting one or more of the characteristic modes of the cavity depends on whether the acoustic losses are, in effect, outweighed by the acoustic gains An assessment of the acoustic gains and losses to be expected in a rocket motor is thus of vital importance to the question of its stability

A number of acoustic sources and sinks associated with rocket motors are illustrated in Fig 1 In general, it appears that the burning process, and thus the burning zone of the propellant itself, is the major acoustic amplifier Hence, the determination of the burning zone acoustic response is recognized to be of special importance Quantitatively, this response is described in terms of the specific acoustic admittance of the burning surface or, alternatively, in terms of the propellant response function (The propellant response function[1-3] is $\tilde{\mu}/\tilde{\epsilon}$, where $\tilde{\mu}$ represents the complex Fourier amplitude describing the dimensionless increment in burning rate, and $\tilde{\epsilon}$ is the corresponding Fourier amplitude describing the dimensionless increment in pressure) The acoustic response of the propellant is considered in more detail later, but for the moment let us turn to a consideration of the acoustic losses

Since instability results unless the acoustic losses are great enough to outweigh the acoustic gains, a determination of the acoustic losses is also vital The expected contributions from all of the loss mechanisms indicated in Fig 1 have not yet been determined Nevertheless, the importance of some of them has now been established on experimental and theoretical grounds Here, we shall mention briefly only three: wall losses, particle damping, and flow and orifice effects

The relative importance of these mechanisms is variable Thus, in T-burners operating at not too high frequencies, it appears that wall loss may sometimes be the dominant loss mechanism This is to be expected because of the large exposed wall areas of T-burners [4 5] On the other hand, little wall area is exposed in the usual rocket motor where such losses would be relatively small, except possibly in the presence of "resonance rods "

In both the T-burner and in rocket motors, however, volume losses have been found sometimes to be of great importance if the product gas contains even a low concentration (say 1%) of particles in the micron-size range Particles of such sizes may be remarkably efficient absorbers of sound There are a number of experimental and theoretical studies bearing directly on this loss mechanism,[6-8] and one of the papers in this issue deals further with the matter [9]

The effects of flow and orifice location must not be neglected These considerations are of particular importance in the reduction of data from T-burner experiments where the rate of growth of oscillation is determined while the propellant is burning, and the rate of decay is often measured after burning has ceased Studies of the acoustic properties of a T-burner type cavity with flow have shown that both the specific admittance of the burning propellant and the propellant response function will be determined incorrectly unless the effects of flow and orifice location are taken into account In fact, the propellant response function (viz , $\tilde{\mu}/\tilde{\epsilon}$) would be assigned too large a value (by approximately $1/\gamma$) [10]

The effects of end nozzles also may be of importance (particularly in practical motors) Such orifices have been considered in several previous works,[11] and Ref 12 is concerned specificially with orifice phenomena

Let us now turn to the determination of the acoustic response of the burning surface T-burner measurements of this quantity (as measured by $\tilde{\mu}/\tilde{\epsilon}$ or by the acoustic admittance) have by now provided us with a considerable amount of experimental data, and one must ask, insofar as it is now possible, how these data compare with the predictions of theory It appears that, for moderately fast burning rates, the experimental data yield propellant response curves (vs frequency) that are rather similar to those obtained from theoretical calculations, viz , a broadly peaked response that falls off at both the low- and high-frequency extremes [5]

For low burning rates, however, Horton and Rice[5] have found propellant response functions that tend to be somewhat larger at low frequencies than theory can easily account for if we assume apparently reasonable values of propellant parameters Because of incomplete knowledge of the losses, the accuracy of the determination of the propellant response function from T-burner experiments is not yet firmly established, so that one does not yet know for certain just how much significance is to be attached to the reduced data Nevertheless, the frequently observed trend toward a relatively large propellant response below about 10^3 cps at low burning rates seems quite impressive This tends to indicate that some very essential process has been omitted from consideration in the construction of the theory

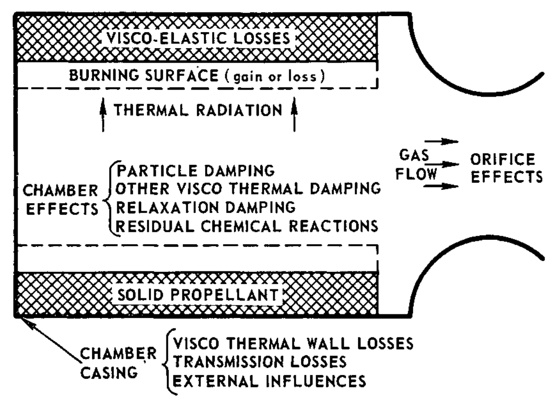

Fig 1 Schematic illustration of acoustic gain and loss mechanisms in a rocket motor

Since the apparent discrepancy seems to show up at low frequencies and low burning rates, our thinking must be oriented toward the recognition of physical processes having at least two distinguishing characteristics: 1) their relative importance should tend to decrease as the burning rate is increased, and 2) they should at least be as sluggish as the process of heat conduction in the solid, which is the slowest time dependent process considered in the previous theoretical treatment Furthermore, since previous theory, which yields the correct steady-state pressure index, is necessarily valid at zero frequency, the effects of the new mechanism which we seek must disappear in the zero frequency limit

The data being considered were collected at a relatively low pressure (200 psi), and it has been suggested that the slow burners under these conditions may be generating significant amounts of gases, which complete their reactions throughout the volume of the chamber rather than near the surface These volume reactions might then be supposed to couple energy into the sound wave A proper investigation requires considerable knowledge of the chemical kinetics of these gases, which is not available We shall not investigate this suggestion here but, rather, shall turn our attention to another, perhaps less specialized mechanism, which appears to have these necessary characteristics just enumerated

II Thermal Radiation and the Acoustic Response of Solid Propellants

One mechanism that can meet the general requirements just discussed involves the heating of the propellant surface by thermal radiation from the burnt gases

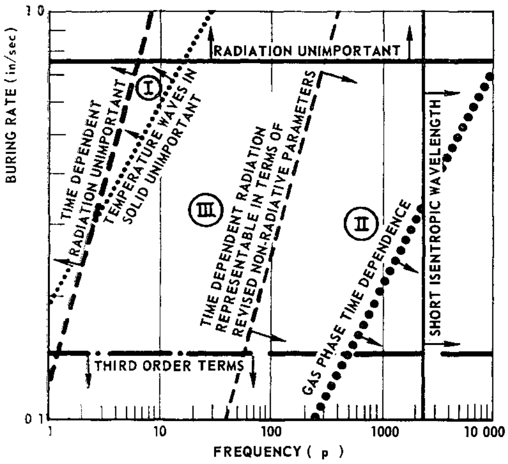

Fig 2 Map illustrating the relative importance of various time dependent processes as a function of burning rate and frequency

An analytical treatment of the effects of thermal radiation on the acoustic response of burning solid propellants has been carried out elsewhere by the authors [13] Here, however, we would like to inquire in more detail into the delineations of the conditions for which radiation effects are to be expected and to present a few new results of the analysis Our initial objective will be that of constructing a map of burning rate vs frequency (cf Fig 2) on which the regions where radiative effects may be important are clearly delineated

1 Magnitude of the Radiation Flux

In order to obtain an indication of whether or not radiation could be important for conditions of interest, we may estimate how large the ratio of radiant heat flux to convective heat flux might be at the burning surface The gas cannot radiate more effectively than a blackbody, and the surface cannot do better than absorb all of the radiation incident on it, so that this ratio will not be greater than

$$\delta_0 = \frac{1\ 37}{r\rho\ C_pT_0}\left(\frac{T_b}{1000°\mathrm{K}}\right)^4 \frac{\mathrm{cal}}{\mathrm{cm^2\text{-}sec}}$$

where r is the linear regression rate of the solid, ρ is the density of the solid, C_p is the constant pressure thermal capacity of the gas, T_b is the temperature of the burnt gas, and T_0 is the temperature of the burning surface of the propellant Thus, we might suppose that the role of radiation will tend to be small if δ_0 is less than about $\frac{1}{10}$ However, a slightly different criterion is actually obtained from the analysis[13]

$$\frac{4j\ 1\ 37}{r\rho\ C_sT_c}\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{T_b}{1000°\mathrm{K}}\right)^4 \frac{\mathrm{cal}}{\mathrm{cm^2\text{-}sec}} < \frac{1}{10}$$

where j is the familiar coefficient describing the sensitivity of burning rate to conditioning temperature Thus, the first boundary line to appear on our map (the heavy horizontal line of Fig 2) is a straight line corresponding to

$$r = \frac{54\ 8}{C_s\ \rho\ T_c}\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{T_b}{1000°\mathrm{K}}\right)^4 \frac{\mathrm{cal}}{\mathrm{cm^2\text{-}sec}}$$

For a hypothetical propellant having the properties listed in the Appendix, r = 0 75 in /sec, and for these conditions, it would seem safe to assume that radiative effects would be quite minor for burning rates faster than 0 75 in /sec For hotter propellants, of course, radiative effects will persist to considerably higher burning rates

2 Frequency Response

Let us now turn to a consideration of the frequency domain that might be associated with the effects of thermal radiation The zero frequency limit is considered first The preceding discussion of course pertains to the steady state, and thus it might seem strange that our previous acoustic theory, in which radiation was ignored, could be correct for the steady state [1-3] This strangeness is only an illusion, however, because that theory was carefully constructed in such a way that in the zero frequency limit the propellant response function correctly approaches the empirically determined pressure exponent n regardless of the mechanisms (such as order of reaction, radiation, etc), which, in fact, contribute to the determination of n Thus, the extension of the previous theory to encompass time dependent thermal radiation effects will result in no modification at sufficiently low frequencies New (radiative) effects may be expected to appear, however, at frequencies sufficiently high that an appreciable phase shift appears in either the radiant emission or in the response of the propellant to the time varying radiant flux

In order to discuss thermal radiation effects, we will first recall that the radiant emission from a gas tends to be an extremely sensitive function of the temperature Furthermore, it is clear that, when the cavity is excited in some acoustic mode, there will be some distribution of burnt gas temperature associated with this mode This temperature field may be resolved into two components: the temperature variations characteristic of the isentropic sound waves, and the temperature variations characteristic of the entropy waves Thus, the temperature field in the cavity is a function also of time, as is the radiant heat flux This contributes to the heat input at the solid propellant surface, and it may be phased so as to enhance or inhibit the heat input arising from thermal conduction at that surface Let us now turn to the question of the characteristic times and consider first the entropy wave field whose wavelength is very short compared to the wavelength of the sound field [14] It will be recalled that entropy waves appear as alternate bright and less bright regions that propagate away from the burning surface at the flow speed [2,14] If it should happen that the absorption length for thermal radiation is comparable to the wavelength of the entropy wave, then the thermal flux originates from a zone whose thickness is comparable to the length of an entropy wave Thus, we may visualize the bright and dim regions of the entropy wave as they pass through this region, giving rise to an oscillating radiant flux This leads us to consider a new characteristic time, namely, the time required for the entropy wave to traverse a distance equal to the radiative absorption length, which may be taken as

$$\tau_r = \frac{1}{\bar{k}\bar{v}} = \frac{1}{(\bar{k}/\bar{p})\bar{m}}$$

We are led, therefore, to the supposition that, as the frequency is increased, the effects of radiation on the acoustic response, which are concealed in the pressure index at zero frequency, will begin to manifest themselves as soon as the "phase shift"

$$\Omega = \frac{\omega}{(\bar{k}/\bar{p})\bar{m}}$$

becomes significant Thus, if we adopt $\Omega = (\frac{1}{10})$ as the value of distinction, it may be supposed that our previous theory, in which radiation was neglected, will remain substantially unmodified whenever

$$r > \frac{10\omega}{(\bar{k}/\bar{p})\rho}$$

and, correspondingly, a new boundary (heavy dashed line of Fig 2) may be drawn on our map For our particular example, we find $r > 0\ 12\ f$ in /sec (with f in cps) The boundaries of the region of interest have now been drawn, and we shall turn our attention toward the nature of the enclosed terrain

3 The Model

In order to obtain a moie detailed picture of the terrain before us, it will be helpful to avail ourselves of the vantage ground provided by a model The model that we shall consider here is basically that of Refs 1 and 3 and is illustrated schematically in Fig 3 There are four zones of essential interest: 1) the zone of the unreacted solid propellant, 2) the solid phase reaction zone, 3) the gas phase zone wherein all gas phase chemistiy is presumed to occur, and 4) the hot thermally radiating product gas Let us now see how the analysis is formulated by considering the description of the various processes that are allowed to occur in each of the four regions

Zone 1 of Fig 3 may easily be disposed of so long as we consider opaque propellants Then thermal radiation may be ignored in this region, and, since the solid is substantially incompressible, it is essential to consider only heat conduction here In this connection, we may recall that there is a characteristic time τ associated with this heat conduction

$$\tau = [\lambda_s \rho / C (\bar{m})^2]$$

(where the subscript s pertains to the unreacted solid) Associated with this time constant is a phase shift $\omega\tau$, and we recognize that the response of this region will be quasi-static for frequencies high enough that $\omega\tau$ is greater than, say, $\frac{1}{40}$ [Here, we use $\frac{1}{40}$ rather than $\frac{1}{10}$ because analysis has shown that it is always $4\omega\tau$ (rather than merely $\omega\tau$), which actually occurs in the time dependent theory (see Refs 1 and 2)]

We may now draw on our map a new contour corresponding to

$$[\omega\lambda\ \rho / C (\bar{m})^2] < \frac{1}{40}$$

that is,

$$r > (80\pi\lambda\ f / C\ \rho_s)^{1/2}$$

Foi our example, we find (for r in inches per second) $r > 1\ 91\ (f/100)^{1/2}$ In the region above this contour (cf the light dotted line of Fig 2), heat conduction in the solid can be considered to be substantially quasi-static

The solid phase reaction zone is rather more complicated, however The problem has been simplified somewhat by treating only "black" propellants for which the radiative absorption length in the solid is negligibly short compared with the thickness of the solid phase reaction zone Thus, insofar as this zone is considered, the radiant flux input is merely added to the heat conduction flux input This means that the quantity $\lambda(\partial T/\partial x)|_{x=0}$, which describes the heat conducted into the solid from the gas in the previous theoretical treatment without radiation, is now to be replaced by the quantity

$$\lambda(\partial T/\partial x)|_{x=0} + q_0$$

where q_0 is the radiant energy absorbed per second by unit area of surface Of course, the calculation of q_0 will lie at the heart of the problem, but the method of its evaluation belongs properly in the consideration of zone 4 of Fig 3 A further simplification is made, following Refs 1 and 2, by supposing that the solid phase reactions are fast enough that the solid phase reaction zone may be collapsed

With respect to the gas region (zone 3 of Fig 3), attention will be confined to low enough frequencies such that the phase shifts associated with heat and mass transfer in this zone are negligibly small Since the relevant characteristic time is (see Ref 1)

$$\tau_h \approx (\lambda\bar{\rho}_0/\bar{m}^2 C_p)$$

we restrict attention to fiequencies such that $\omega\tau_h < \frac{1}{10}$, that is,

$$\bar{m} > (10\lambda\rho_0\omega/C_p)^{1/2}$$

or

$$r > (1/\rho)(20\pi f\lambda\rho_0/C_p)^{1/2}$$

For our example, this leads to $r > 0\ 22\ (f/1000)^{1/2}$ in /sec Thus, we draw a new contour line on our map (cf heavy dotted line of Fig 2) which portrays the boundary between an upper region, which we shall consider here, in which the gas phase can be treated as "fast" and a lower region in which it becomes important to describe the transfer of mass and the conduction of heat in the gas phase by the time dependent conservation equation

Let us now turn to the consideration of zone 4 of Fig 3 This is the region that constitutes the source of the radiant flux, and it encompasses essentially all of the volume of the cavity Our task is that of evaluating the time dependent radiative flux that leaves the left surface of this region and is thus incident on the propellant surface In order to make the problem tractable, we have made several approximations

First, it is assumed that the gas may be regarded as having the radiative properties of a gray body Thus, the emission from each element of volume is assumed to be given by an absorption coefficient k multiplied by the emission which would characterize the element if it were a blackbody, i e, by $k\sigma_* T^4$, where σ_* is the Stefan-Boltzmann constant The absorption coefficient is, of course, proportional to the gas density The significance of this approximation would seem quite difficult to determine accurately, although we suppose that it would be quite reasonable, particularly if one used an appropriate "effective" gas temperature

Second, the problem treated is strictly one-dimensional, and heat loss to the walls of the chamber is ignored In fact, the mechanism of heat conduction in zone 4 of Fig 3 is entirely neglected on the basis that the energy flux aiising from the mean flow of the gas is by far the more important energy transport mechanism here This neglect of heat conduction may tend to become serious, however, unless the radiative absorption length is small compared with the length of the cavity Accordingly, we shall restrict our at-

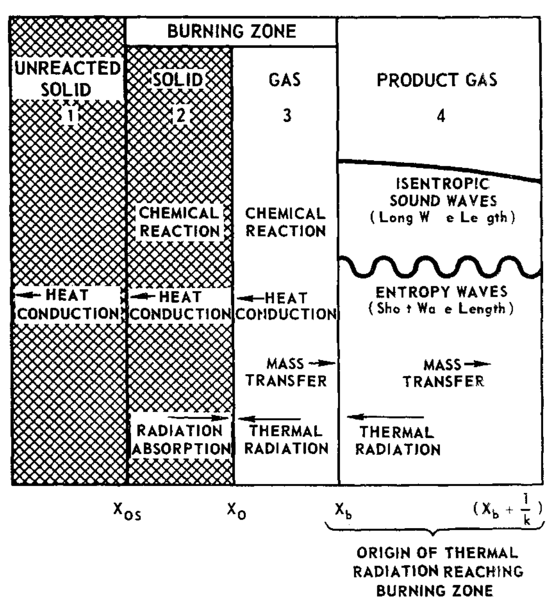

Fig 3 Schematic model of the burning propellant

tention to conditions for which the absorption length for radiation is small compared with the sound field wavelength and thereby utilize a third approximation, which greatly simplifies the analysis. This third approximation takes advantage of the fact that when the radiative absorption length is very small, compared with the wavelength λ of the isentropic sound wave, the pressure may be taken to be uniform over the region from which the radiation reaching the burning surface originates As a result, the extent of the terrain, which we shall explore, has now been further limited to include only that region for which $(1/\bar{k}\lambda)$ is less than, say, $\frac{1}{10}$ For mapping purposes, this criterion is more conveniently stated in terms of frequency, $f < (\frac{1}{10})\bar{k}c$, where $c \equiv (\gamma\bar{P}/\bar{p}_b)^{1/2}$ For our example, this leads to $f < 2\ 38 \times 10^3$ cps, which is shown as the heavy vertical line of Fig 2

Further, in order to make the analysis relatively tractable, we have treated the effect of radiation as a perturbation on the zero-radiation fields (using a quantity similar to δ_0 as the expansion parameter) Thus, if the radiative effect becomes too large, the present theoretical treatment will become invalid, and the extent of the explorable terrain is further limited If we adopt for our criterion of validity the condition that the product of the first- and second-order terms be less than $\frac{1}{10}$ (this product is presumed to be a reasonable estimate of the order of magnitude of the third-order terms that were dropped), we obtain the criterion

$$r > 1\ 7\left(\frac{\sigma_* T_b{}^4}{\rho\, C_p T_b}\right)\left[j^2\left(1 + j\,\frac{C_p T_b}{C\, T}\right)\right]^{1/3}$$

For the hypothetical propellant, we find $r \gtrsim 0\ 125$ in /sec, and this determines the lower of the two horizontal lines of Fig 2

III Discussion

At this point, the regions of present interest have been mapped, and the boundaries within which the theoretical treatment should be expected to apply are shown in Fig 2 It appears that the present theory will apply to a relatively large part of this map

As might be expected, the result of solving the equations describing even the highly idealized model, which has been described, leads to unwieldy expressions, and there would seem to be little point in reproducing them here (The entire development is given in Ref 13) The several processes interact in such an intricate way that it is difficult to understand the problem well enough to predict the result of a given calculation in advance Nevertheless, it is possible to recognize some salient features

First, we may note that there is a region of terrain (I) on the map of Fig 2 in which the time dependence of heat conduction and mass transfer is relatively unimportant, and in which radiative transfer is the time dependent mechanism of major importance

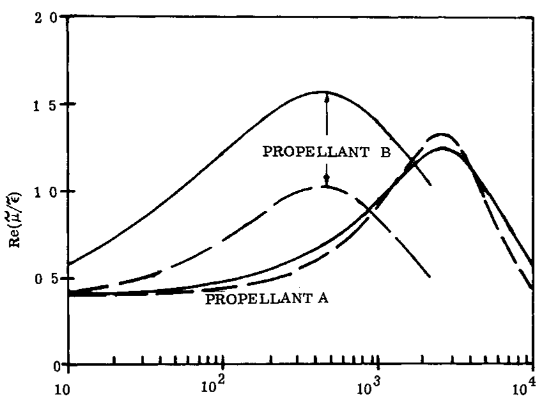

Fig 4 Example of the effect of radiation on the acoustic response of two hypothetical propellants The dashed lines are those obtained when radiation is ignored

If we had neglected radiative effects, the propellant response would have been given by $\tilde{\mu}_0/\bar{\epsilon} = n$ in this region When radiation is taken into account however, the propellant response may be significantly affected (cf Fig 4) Along the left and top boundaries of region I, the response function approaches its steady-state value, but as we move away from these boundaries, Ω and δ both increase and, in general, so does the real part of $\tilde{\mu}_0/\bar{\epsilon}$ (for $j > 0$) This behavior can be inferred from the steady-state treatment of the radiative effect on pressure response (cf Ref 13) where it is shown that, at zero frequency, radiation tends to depress the pressure response of a propellant with positive temperature sensitivity Thus, the radiation effect at zero frequency corresponds to "negative feedback " But as the frequency is increased from zero, a phase shift develops, and the pressure response tends to increase It is interesting to note, parenthetically, that, in region I, $\tilde{\mu}_0/\bar{\epsilon}$ scales with Ω, i e , with $f/\bar{m}$ if the other quantities in the forementioned expression are not varied

There is another region of particular interest on our map It turns out from the analysis of Ref 13 that, for a large Ω [say $\Omega > 5$ for which $r > 2\ 48\ (f/1000)$, which is shown as the light dashed line in Fig 2], the time dependent response of the propellant is described by the same equations that would pertain in the absence of radiation, except that the values of n and j which appear are no longer the steady-state values Instead, one finds that one must use modified values n' and j', which are primarily functions of n, j, $\bar{m}$, T_b, and T This region is denoted as II on Fig 2 Here, the general nature of the predictions of the previous "radiationless" treatment remain valid,[1-3] e g , the amplifying ability of propellants will still tend to be a broad banded function of frequency However, the appropriate values of n and j are not the steady-state values, but the n' and j' calculated from the radiation theory (cf Ref 13) The practical effect of this observation depends, of course, on how significantly n' and j' differ from n and j It turns out that n' easily may be made substantially greater than n, so that many propellants that would be acoustic dampers in the regions where time radiative effects are quasistatic may become sound amplifiers in region II

The time dependence of both the radiative and heat conduction mechanisms are also important in the transition region III Here the interplay between these two processes is rather complex, and we have been forced to rely entirely on numerical methods for exploration Illustrations of the kinds of results obtained are given in Ref 13 Figures 4 and 5 show the results of several new calculations for two hypothetical propellants, with properties listed in the Appendix, and further illustrate the point that time dependent radiation effects may contribute toward the production of rather large propellant response in the low-to-moderate frequency regime

In Fig 4, "propellant" A is characterized by a burning rate, which places it near the upper boundary of the map in Fig 2 Thus, the propellant response is not greatly affected by radiation "Propellant" B, on the other hand, has a slow burning rate and falls near the bottom boundary of the map where the effect of radiation is quite large It is interesting to note that, if we had not considered radiation, the maximum response of propellant B would be less than that of propellant A, but that, when radiative effects are considered, the situation is reversed, and the maximum response of propellant A is found to be less than that of propellant B

Figure 5 illustrates the dependence of the response function on the reciprocal absorption length k As the discussion has suggested, the response at the higher frequencies is independent of k, and the influence of time dependent radiation becomes evident at lower and lower frequencies as the absorption length is increased (i e , as k is decreased) Figure

5 also illustrates the effect of a change in burning rate alone This figure has a particular significance when we note that the abscissa is f/r^2 and recall that, if radiation is neglected, and if heat conduction in the solid were the only important nonadiabatic process, then there would be only a single curve (the lower one in the figure) §

IV Concluding Remarks

Particular attention has been devoted to the discussion of conditions for which absorption of thermal radiation by the burning surface should be expected to be of importance to the acoustic response of solid propellants The conditions under which the time dependence of radiation should modify the acoustic response are summarized, for one example, by Fig 2 Similar maps might easily be drawn for any desired propellant if the values of the relevant parameters characterizing the propellant were known It will be noted that there can be a rather large range of burning rates and frequencies for which time dependent radiative effects should appear, and we have presented results of several calculations, Figs 4 and 5 (using the theoretical expressions of Ref 13), which illustrate these effects Further, it seems highly significant that thermal radiation may produce such large effects even for the rather low effective gas temperature (2400°K), which was used for our example

Appendix

In constructing the map in Fig 2, we consider a propellant and its product gas with the following properties:

Gas:

$T_b = 2400°K$ $\quad C_p = \frac{1}{3}$ cal/g-°K

$T_0 = 600°K$ $\quad k_b = \frac{1}{4}$ cm^{-1}

$\gamma = 1\ 2$ $\quad \bar{P} = 200$ psi

$\rho_b = 2\ 0\ (10)^{-3}$ g/cm^3 $\quad \sigma_* = 1\ 37\ (10)^{-12}$ cal/cm-sec-°K^4

Propellant:

$T = 300°K$ $\quad C = \frac{1}{3}$ cal/g-°K

$j = 1$ $\quad \rho = 1\ 6$ g/cm^3

$\lambda = 5(10)^{-4}$ cal/cm-sec°K

"Propellants" A and B (see Figs 4 and 5) are characterized by the forementioned, except where noted below, and by the following additional parameters Additional nomenclature is that of Refs 1–3

A:

$T_0 = 1200°K$ $\quad \alpha = -1$

$A = 50{,}000$ cal/mole $\quad h_v = 200$ cal/g

$n = 0\ 4$ $\quad \rho = 1\ 62$ g/cm^3

$j = 0\ 4$ $\quad \gamma = 1\ 22$

$r = 0\ 545$ in /sec

$\rho_b = 2\ 13(10)^{-3}$ g/cm^3

B:

$T_0 = 600°K$ $\quad \alpha = -1$

$A = 50{,}000$ cal/mole $\quad h_v = 200$ cal/g

$n = 0\ 4$ $\quad \rho = 1\ 62$ g/cm^3

$j = 1$ $\quad \gamma = 1\ 22$

$r = 0\ 195$ in /sec

$\rho_b = 2\ 13\ (10)^{-3}$ g/cm^3

§ In that case, the response function depends on frequency and burning rate only through the ratio f/r^2 other parameters being held fixed Actually, of course, in order to alter the burning rate, we would have to change at least one of the remaining parameters, say the solid phase activation energy The required change is small, however, and does not in itself alter the response function appreciably

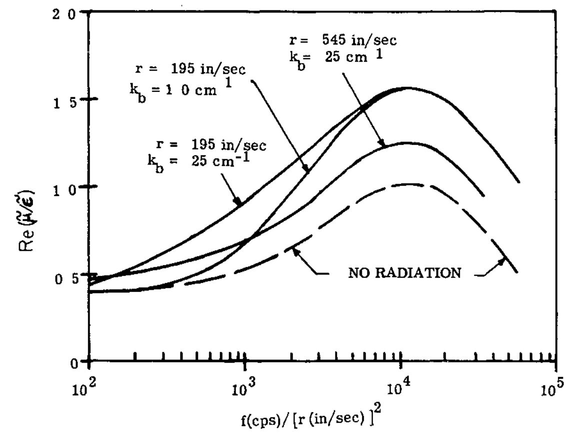

Fig 5 An example of effect of radiation absorption length and burning rate on acoustic response The dashed line is obtained when radiation is ignored

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